The Millikan Oil-Drop Experiment

Abstract

In this experiment we repeated Millikan's oil-drop experiment which was carried out in 1909 and which was partly responsible for his winning the Nobel prize in 1923. We succeeded in calculating the charge on the electron.

Introduction

With the apparatus supplied it was possible to repeat Millikan's very important experiment of 1909 in which he established the discreteness of the electronic charge and determined its magnitude. The charge carried by the electron is a fundamental physical constant. Techniques employed prior to this experiment allowed scientists to deduce only the average electron charge. Millikan was the first to show that this quantity was discrete or single valued. For this work and that on the photoelectric effect, he was awarded the Nobel prize for physics in 1923.
In this experiment a small, charged drop of oil was observed in a closed chamber between two horizontal, parallel plates. By measuring the velocity of fall of the drop under gravity and its velocity of rise when the plates are at a high electrical potential difference, data was obtained from which the charge on the drop was computed.

Theory

When no electrical field is applied the oil drop is subject to the force of gravity which is opposed by the resistance of the viscous fluid and the buoyancy of the drop.

$$mg = 6\pi a\eta v_g + 4/3\ \pi a^3\alpha_l g \qquad (1)$$

Here the mass of the oil drop is m, the first term on the right is the viscous force dependant on the terminal velocity that the oil drop reaches, v_g, the radius of the sphere of the drop, a, and the viscosity of the fluid, η, The second term on the right is the buoyancy where α_l is the density of air.

When there is an applied Electric field, E, acting on a drop with charge, q, the equation for the constant upward velocity, v_E, is:

$$mg + 6\pi a\eta v_E = Eq + 4/3\ \pi a^3\alpha_l g \qquad (2)$$

The term associated with the buoyancy can be eliminated between equations (1) and (2) and writing the electric field E=V (voltage applied)/d (distance between plates) we obtain an expression for the charge on the drop, q, [1]

$$q = \frac{6\pi a\eta d}{V}(v_E + v_g) \quad (3)$$

So that for the same drop and applied voltage a change in the charge q results only in a change in the velocity v_E and

$$\Delta q = c\Delta v_E \quad (4) \quad \text{with } c = 6\pi a\eta d/V.$$

If the quantities v_E and v_g are determined experimental, all the quantities in equation (3) are known with the exception of the radius of the drop, a, which is given by

$$a = \sqrt{\frac{9\eta v_g}{2g\alpha}} \quad (5) \qquad [1].$$

Using this expression in equation (3) and multiplying the viscosity by a correction factor $(1+b/pa)^{-1}$, with $b = 6.17 \times 10^{-6}$ and p the barometric pressure in cm of mercury, yields the corrected charge on the drop [1]:

$$q = \frac{6\pi d}{V}\sqrt{\frac{9\eta^3}{2\alpha g}}(1+b/pa)^{-3/2}(v_E+v_g)\sqrt{v_g} \quad (6)$$

Experimental Procedure

The apparatus, shown in figure 1, was aligned and focussed as described in the experimental guidelines [1]. Oil was introduced into the system and observed through the microscope. Some time was invested in learning how to manipulate the drops using the applied electric field.

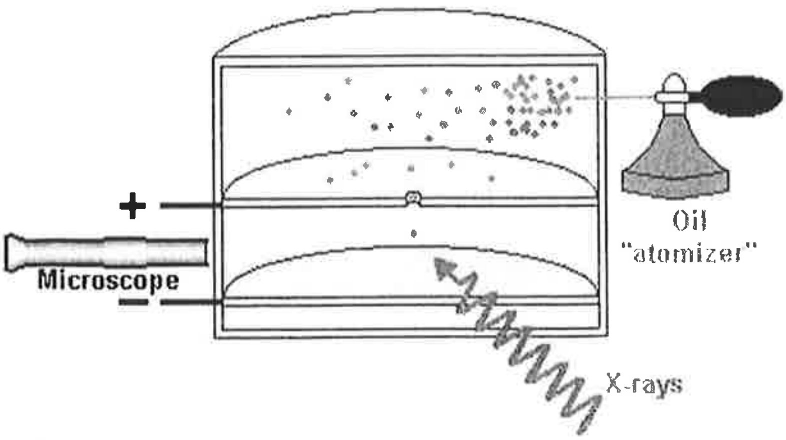


Figure 1: Experimental Apparatus.

The barometric pressure was recorded and found to be p = 76 cm of mercury. The microscope was calibrated and the distance represented by each division on the microscope scale was found. The plate separation, d, was found to be 5mm and the voltage applied to the plates, V, was 340 V.
When no electric field was applied, preliminary measurements were made by timing the movement of a drop over different distances. Subsequently, the velocity, v_g, of a particular drop was found by timing the motion of the drop over a specific distance (d2=0.0005m) on several occasions with no applied electric field. A similar procedure was carried out for the motion of the same drop in the applied field in order to determine v_E. Having determined the radius of the drop using equation (5), the charge on the drop was found using equation (6). The value of the charge on the electron (e) was then used to determine the integer number of multiples of the charge on the electron that were carried by the oil drop. Using this integer an experimental value for e was obtained.

Results

Figure 2 shows a plot of time taken against the distance travelled for an oil drop moving in the absence of an applied electric field. The velocity, v_g, can be seen to be constant as the graph is linear and the velocity determined from the slope of the graph was found to be 9.3 x10-5 +/- 0. 3 m/s.

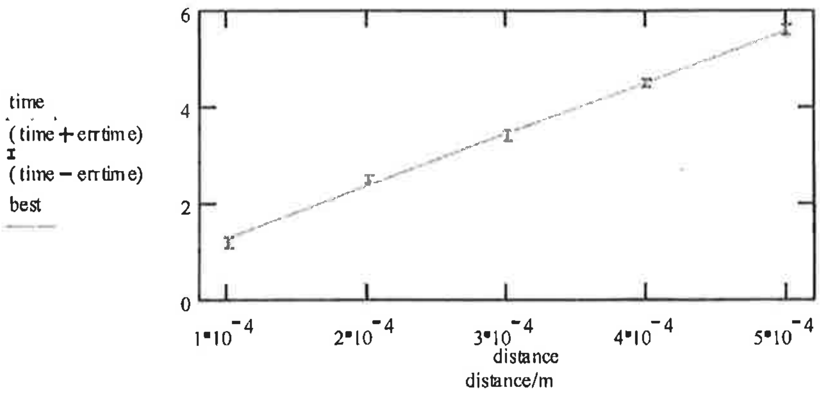


Figure 2. Time versus distance.

Subsequently, the motion of a drop over a specific distance (d2=0.0005m) was monitored on 10 occasions both in the presence and absence of the electric field. The mean fall time was used to determine v_g and the mean rise time, in the presence of the applied electric field, was used to determine v_E.

v_g was found to be 8.921E-5*m*s^-1 +/- 7.936E-6*m*s^-1 whilst
v_E was found to be 1.372E-4*m*s^-1 +/- 1.019E-5*m*s^-1.

The radius of the drop was then calculated using equation (5), this was found to be 9.2 +/- 0.4 x10^-7m. This value was then used to determine the total charge on the drop, q=9.441E-19. This is essentially 6 times the charge on the electron, giving an experimental value for the charge on the electron of 1.573E-19 +/- 1.172E-20 C, which is consistent with the accepted value.

Discussion

The value obtained for the charge on the electron is consistent with the accepted value of 1.602 176 53(14) × 10^{-19} C. The experiment could be improved if the apparatus was easier to use. Human error is responsible for the difference between the experimental value and the accepted value.

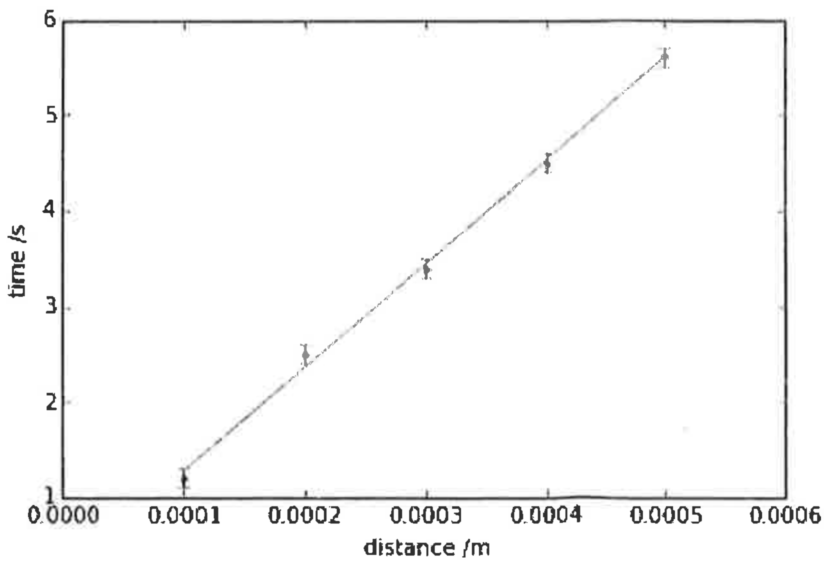
Conclusion

References:
[1] UCD 2nd Year Physics Laboratory Manual 2005.

iPython Notebook Millikan Oil Drop Appendix

In [1]:
```
import numpy as np
import astropy.units as units
import astropy.constants as cnts

#  data from experiment
distance = np.array([1,2,3,4,5])*.0001*units.meter
time=np.array([1.2,2.5,3.4,4.5,5.6])*units.second
errtime=np.ones(5)*.1*units.second
```

In [2]:
```
import matplotlib.pyplot as plt
%matplotlib inline
#This part is the plot with error bars .value removes the units
x=distance.value
y=time.value
erry=errtime.value
#Fitting for straight line
from scipy.optimize import curve_fit
def func(x, a, b):
     return a*x + b

popt, pcov = curve_fit(func, x, y, sigma=erry)

best=popt[0]*x+popt[1]

plt.errorbar(x,y,yerr=erry,fmt='.')
plt.plot(x,best,'r')
plt.ylabel('time /s')
plt.xlabel('distance /m')
plt.show()
```



The error on the velocity (which is 1/m) is $\frac{\Delta m}{m^2}$.

In [3]:
```
errvel=np.sqrt(pcov[0,0])/popt[0]**2
print("Velocity is {0:.3e}+/-{1:.3e} m/s.".format(1/popt[0],errvel))
```

```
Velocity is 9.259e-05+/-2.619e-06 m/s.
```

This part is for the determination of v_E and v_g.

In [4]:
```
dist=.0005
riset=[3.41,3.461,3.852,3.359,3.348,3.898,4.172,3.633,3.57,3.742]
fallt=[5.762,5.43,5.16,5.652,6.148,6.313,5.711,4.781,5.051,6.039]
averrise=np.mean(riset)
averfall=np.mean(fallt)
stdrise=np.std(riset)
stdfall=np.std(fallt)
```

The uncertainty on the sample mean is related to the standard deviation by: $S = \frac{\sigma\sqrt{N}}{\sqrt{N-1}}$.

In [5]:
```
errrt=stdrise*np.sqrt(len(riset))/np.sqrt(len(riset)-1)
errft=stdfall*np.sqrt(len(fallt))/np.sqrt(len(fallt)-1)
```

The rise and fall velocities are calculated from the mean rise (fall) times: $velocity = \frac{distance}{time}$, the uncertainty on the velocity is calculated from the uncertainty in the time $\Delta velocity = \frac{distance\Delta time}{time^2}$.

In [6]:
```
ve=dist/averrise
vg=dist/averfall
errve=errrt*dist/averrise**2
errvg=errft*dist/averfall**2
print("VE is {0:.3e}+/-{1:.3e} m/s.".format(ve,errve))
print("Vg is {0:.3e}+/-{1:.3e} m/s.".format(vg,errvg))
```

```
VE is 1.372e-04+/-1.019e-05 m/s.
Vg is 8.921e-05+/-7.936e-06 m/s.
```

In [7]:
```
#In order to calculate the radius of the drop from equation 18.8, need  vg
,g, density of oil (alpha) and the viscosity of air (eta)
vgu=vg*units.meter/units.second
errvgu=errvg*units.meter/units.second
eta=1.85e-5*units.newton*units.second/units.meter**2
g=9.8*units.meter/units.second**2
alpha=890*units.kg/units.meter**3
a=np.sqrt(9*eta*vgu/(2*g*alpha))
erra=np.sqrt(9*eta/(2*g*alpha))*.5*errvgu/np.sqrt(vgu)

print("Drop radius is {0:.3e} +/- {1:.3e} .".format(a.to('m'),erra.to('m')
))
```

```
Drop radius is 9.228e-07 m +/- 4.104e-08 m .
```

In [8]:
```
#Calculation of charge q, according to equation 18.10
veu=ve*units.meter/units.second
errveu=errve*units.meter/units.second
V=340*units.Volt

sepp=.005*units.meter   #Plate separation
b=6.17e-6
p=76/units.meter  #These last two are not in mks units
q=((6*np.pi*sepp)/V)*np.sqrt(9*eta**3/(2*alpha*g))*(1+b/(p*a))**-1.5*(veu+
vgu)*np.sqrt(vgu)


errqsq=((1+b/(p*a))**-1.5*np.sqrt(vgu)*errveu)**2
errqsq=errqsq+((1+b/(p*a))**-1.5*.5*veu*errvgu/np.sqrt(vgu))**2
errqsq=errqsq+((1+b/(p*a))**-1.5*1.5*errvgu*np.sqrt(vgu))**2
errqsq=errqsq+(1.5*(1+b/(p*a))**-2.5*(b*erra/(p*a*a))*np.sqrt(vgu)*(veu+vg
u))**2
frontfactor=(((6*np.pi*sepp)/V)*np.sqrt(9*eta**3/(2*alpha*g)))**2

errq=np.sqrt(frontfactor*errqsq)

print("Drop charge is {0:.3e} +/- {1:.3e} .".format(q.to('C'),errq.to('C')
))
```

```
Drop charge is 9.441e-19 C +/- 7.029e-20 C .
```

Next part is to calculate the number of charges:

In [9]:
```
chargee=1.6022e-19
N=(q.value/chargee)
N
```

Out[9]:
```
5.892413687224733
```

This suggests that there are 6 charges on the drop.

In [10]:
```
#Experimental Value of the charge on electron
eexp=q/6
erreexp=errq/6
print("The experimental value for the charge on the electron is {0:.3e} +/
- {1:.3e} .".format(eexp.to('C'),erreexp.to('C')))
```

```
The experimental value for the charge on the electron is 1.573e-19 C +/- 1
.172e-20 C .
```

In [10]: